Consumption and Labor Income Risk, Aggregation and Business Cycles

Jose Ignacio Lopez

UCLA

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Aggregate Data reveals a *systematic* deviation between the Marginal Rate of Substitution between Consumption and Leisure (MRS) and the Marginal Product of Labor - the so-called "Labor Wedge"

This wedge matters for:

- **Business Cycles:** It is highly counter-cyclical - households get discourage from working as if during recessions labor taxes were higher.
- **Long Run:** Differences in Labor Supply (US vs Europe)
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- I solve a model in which the aggregation wedge replicates the counter-cyclical pattern. (The key mechanism is the various labor supply elasticities induced by productivity and wealth distribution)
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In this paper I show that the aggregation wedge can explain (up to a degree) to the "Labor Wedge"

- I solve a model in which the aggregation wedge replicates the counter-cyclical pattern. (The key mechanism is the various labor supply elasticities induced by productivity and wealth distribution)
- Using micro data I calculate the aggregation wedge and show that it is consistent with both the long and short run properties of the Labor Wedge

- Explanations:

On Aggregation


Heterogeneous Agent and Aggregate Shocks

Road Map

- Model with productivity shocks (both household-specific and aggregate)
- Aggregation Properties
- The Role of Risk Sharing
- Calibration and Solution Method
- Simulation and statistics of the model
  - This model delivers a counter-cyclical labor wedge and it has novel dynamics for aggregate labor
- Describe micro data
- Direct Calculation of the Aggregation Wedge
1. Large number of agents who have the same preferences over consumption and disutility of work

2. Agents are ex-post heterogeneous, depending on the history of realizations of household-specific productivities.

3. An event $s$ is defined over the possible states of idiosyncratic ($\varepsilon$) and aggregate shocks ($Z$). A history $s^t$ is the collection of all realizations up to period $t$.

4. There is a representative firm that produces the final good with a constant returns to scale technology.
Households

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^i (s^t)^{1-\gamma}}{1-\gamma} - \alpha h_t^i (s^t)^{1+\frac{1}{\eta}} \right)
\]

\[
c_t^i (s^t) = W_t (s^t) \varepsilon_t^i (s^t) h_t^i (s^t) + \Delta a_t^i (s^t)
\]

\[
\log (\varepsilon_t) = \rho_\varepsilon \log (\varepsilon_{t-1}) + \varepsilon_\varepsilon
\]

\[
\varepsilon_\varepsilon \sim N \left( 0, \sigma^2_\varepsilon \right)
\]
\[
\max_{\{K_t(s^t), H_t(s^t)\}} Z_t(s^t) K_t(s^t)^\alpha H_t(s^t)^{1-\alpha} - W_t(s^t) H_t(s^t) - R_t(s^t) K_t(s^t)
\]

\[
\begin{align*}
MPL_t &= \alpha \frac{Y_t(s^t)}{H_t(s^t)} = W_t(s^t) \\
MPK_t &= (1 - \alpha) \frac{Y_t(s^t)}{K_t(s^t)} = R_t(s^t)
\end{align*}
\]

\[
\log(Z_t) = \rho_z \log(Z_{t-1}) + \epsilon_z \\
\epsilon_z \sim N(0, \sigma_z^2)
\]  

(1)
If we aggregate the labor-leisure of each agent:

\[ Ah_i (s^t)^{1/\eta} = W_t (s^t) \varepsilon_i (s^t) c_i (s^t)^{-\gamma} \]

\[ L_t = \sum_i h_i \quad C_t = \sum_i c_i \]

\[ \hat{A}L_t^{1/\eta} (s^t) C_t^\gamma (s^t) = \left[ \sum_i \frac{\varepsilon_i (s^t)^{\eta}}{c_i (s^t)^{\eta\gamma}} \right]^{1/\eta} W_t (s^t) \]
There are two cases for which we can characterized consumption allocation

**With Complete markets (Complete Insurance)**

\[ AW^C_t = (1 - \tau^C_t) = \left[ \sum_i \epsilon_{i,t}^\eta \right]^{\frac{1}{\eta}} \]

**With Financial-Autarky (No-Insurance):**

\[ AW^{FA}_t = (1 - t^{FA}_t) = \left[ \sum_i \epsilon_{i,t}^{1+\eta \gamma} \right]^{\frac{1}{\eta}} \]
The Aggregation Wedge and the degree of Risk Sharing

\[ \tau_t^c = \frac{1-\eta}{2} V_t(\varepsilon) \quad \tau_t^{NI} = \frac{(1-\gamma)}{1+\eta\gamma} \tau_t^c \]

Figure 1. Simulated Hours Worked under Complete Markets and Financial Autarky
The Aggregation Wedge with Partial Insurance (borrowing constraint)

\[ c_t^i (s^t) + a_{t+1}^i (s^t) = W_t (s^t) \varepsilon_t^i (s^t) h_t^i (s^t) + (R_t (s^t) + 1 - \delta) a_t^i (s^{t-1}) \]

\[ a_{t+1}^i (s^t) \geq 0 \]

- This problem is hard to solve because the decision of each household depends on the whole history of individual shocks. The entire wealth distribution is a state variable.
- I will use a perturbation method (Judd (1998) and direct aggregation - Preston and Roca (2007), Den Haan and Ocaktan (2009) -
We can use a perturbation method to solve each agent’s problem and then use direct aggregation:

Advantages:
- It can handle large (infinite) number of agents
- Idiosyncratic shocks can take a continuum of values

The borrowing constraint is not differentiable: I replace it by a penalty function (Kim, Kollmann and Kim (2010) \( b = 0 \))

\[
\log \left( \frac{a_{t+1} + b}{\bar{a}_{ss} + b} \right) - \frac{a_{t+1} - \bar{a}_{ss}}{\bar{a}_{ss}}
\]

This particular penalty function also serves to induce stationarity of asset holdings
We guess a law of motion for aggregate variables:

\[ K_{t+1} = F_K \left( Z_t, K_t, M_{aet}, M_{a_1^2}, \Pi^1 \right) \]

Based on these law of motion, we find an optimal decision rule for each household

\[ a_{t+1}^i = F \left( a_{i,t}, e_{i,t}, e^z_{i,t}, e_{i,t}, Z_t, K_t, M_{aet}, M_{a_1^2}, \Theta^1 \right) \]

Aggregation of the individual-level policy functions will imply a new law of motion for the aggregates:

\[ \sum_i a_{t+1}^i = K_{t+1} = F_K \left( Z_t, K_t, M_{aet}, M_{a_1^2}, \Pi^2 \right) \]
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<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\beta$</td>
<td>Discount Factor</td>
<td>0.98</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation Rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$A$</td>
<td>To Target $h=1/3$</td>
<td>2.46</td>
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<td>$\gamma$</td>
<td>IES</td>
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<td>$\eta$</td>
<td>Elasticity Labor Supply</td>
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<tr>
<td>$\phi$</td>
<td>Penalty Function</td>
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<td>$\rho_\varepsilon$</td>
<td>Agent-specific shock persistence</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma_\varepsilon(z)$</td>
<td>Agent-specific shock stdv</td>
<td>0.12 (0.21)</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Aggregate shock persistence</td>
<td>0.95</td>
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<tr>
<td>$\sigma_z$</td>
<td>Aggregate shock stdv</td>
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<td>------------------</td>
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<tr>
<td><strong>Volatility</strong></td>
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<tr>
<td>Y</td>
<td>1.34</td>
<td>1.34</td>
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<td><strong>Volatility relative to Output</strong></td>
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<tr>
<td>C</td>
<td>0.57</td>
<td>0.41</td>
</tr>
<tr>
<td>L</td>
<td>0.98</td>
<td>0.52</td>
</tr>
<tr>
<td>K</td>
<td>0.10</td>
<td>0.29</td>
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<td><strong>Correlations</strong></td>
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<td>corr(C,Y)</td>
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<tr>
<td>corr(K,Y)</td>
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<td>corr(LW, L)</td>
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<tr>
<td>corr(LW, Y)</td>
<td>0.56</td>
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Quarterly data. 1980:Q1-2009:Q2
### Aggregate Law of Motion Coefficients

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<th>RBC</th>
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<th></th>
<th></th>
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<tr>
<td></td>
<td>K</td>
<td>L</td>
<td>K</td>
<td>H</td>
<td>MAE</td>
<td>MA2</td>
<td>L</td>
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<td>0.9869</td>
<td>-0.0178</td>
<td>0.112</td>
<td>-0.2683</td>
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<td>(z(-1))</td>
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<td>0.3796</td>
<td>0.7137</td>
<td>0.4297</td>
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<td>0.9476</td>
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<tr>
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<td>-0.0223</td>
<td>0.8211</td>
<td>0.1685</td>
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<tr>
<td>(MA2(-1))</td>
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<td>-</td>
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<td>0.0022</td>
<td>0.0049</td>
<td>0.9109</td>
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<tr>
<td>(k(-1)k(-1))</td>
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<td>(z(-1)z(-1))</td>
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<td>-0.0035</td>
<td>-0.0219</td>
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<td>(MAE(-1),z(-1))</td>
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<td>(MA2(-1),MAE(-1))</td>
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<td>(MAE(-1),MAE(-1))</td>
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<td>(MA2(-1),MA2(-1))</td>
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<td>0</td>
<td>0.0002</td>
<td>0</td>
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</tbody>
</table>
Using data on wages and consumption we can calculate the aggregation wedge without assuming any financial structure.

Data Sources: CPS and CEX
Figure 2. Consumption and Wage Inequality in the US
The Labor Wedge and Aggregation Wedges in the Long Run

- Labor Wedge
- Aggregation Wedge

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## Business Cycle Statistics

<table>
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<tr>
<th>IES=1</th>
<th>Frisch Elasticity</th>
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</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

### Correlations

- **With Output**: 
  - 1: -0.13
  - 1.5: -0.19
  - 2: -0.21
  - 2.5: -0.23

- **With Labor**: 
  - 1: -0.23
  - 1.5: -0.28
  - 2: -0.31
  - 2.5: -0.32

- **With the LW**: 
  - 1: 0.22
  - 1.5: 0.26
  - 2: 0.29
  - 2.5: 0.30
The Labor Wedge and Aggregation Wedges during the Business Cycle

Labor Wedge
Aggregation Wedge

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Conclusions. Thanks
IRF. Relax Borrowing Constraint
I re-define the stochastic process of idiosyncratic shocks as follows:

\[ \log (\varepsilon_t) = \rho_\varepsilon \log (\varepsilon_{t-1}) + Z_t^\lambda \varepsilon_\varepsilon \]  

(2)  

If \( \lambda = 0 \), the variance of idiosyncratic shocks is constant over the cycle.  

I set \( \lambda \) to match the fact that during a deep recession the standard deviation of idiosyncratic shocks doubles. \( \lambda = \frac{\log 2}{\log (0.986)} = -49.16. \)
\[ a_{t+1}^i = \theta_0 + \theta_1 a_t^i + \theta_2 \varepsilon_t^i + \theta_3 Z_t + \theta_4 K_t + \theta_5 M_{aet} + \theta_6 M_{a_t^2} + \theta_7 e_t^i + \theta_8 \varepsilon_t^{iz} + \theta_9 a_t^{iz} + \theta_{10} a_t^i \varepsilon_t^i + \theta_{11} \varepsilon_t^{iz} + \theta_{12} Z_t a_t^i + \theta_{13} Z_t \varepsilon_t^i + \theta_{14} Z_t^2 + \theta_{15} K_t a_t^i + \theta_{16} K_t \varepsilon_t^i + \theta_{17} K_t Z_t + \theta_{18} K_t^2 + \theta_{19} a_t^i M_{aet} + \theta_{20} \varepsilon_t^i M_{aet} + \theta_{21} Z_t M_{aet} + \theta_{22} K_t M_{aet} + \theta_{23} M_{aet}^2 + \theta_{24} a_t^i M_{at^2} + \theta_{25} \varepsilon_t^i M_{a^2} + \theta_{26} Z_t M_{at^2} + \theta_{27} K_t M_{at^2} + \theta_{28} M_{aet} M_{at^2} + \theta_{29} M_{at^2}^2 + \theta_{30} \varepsilon_t^{iz} + \theta_{31} e_t^i e_t^i + \theta_{32} \varepsilon_t^{iz} + \theta_{33} a_t^i e_t^i + \theta_{34} a_t^i e_t^{iz} + \theta_{35} \varepsilon_t^i e_t^i + \theta_{36} e_t^{iz} + \theta_{37} Z_t e_t^i + \theta_{38} Z_t e_t^{iz} + \theta_{39} K_t e_t^i + \theta_{40} K_t e_t^{iz} + \theta_{41} M_{aet} e_t^i + \theta_{42} M_{aet} e_t^{iz} + \theta_{43} M_{at^2} e_t^i + \theta_{44} M_{at^2} e_t^{iz} \]