Menu Costs and Phillips Curve
by
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Menu costs and repricing decisions

- Micro foundation of sticky prices: menu costs models
- Two central issues for menu cost models
  1. Firm’s decision to reprice or not (timing of the updating)
  2. Size of price changes
- Do Calvo models do a good job describing pricing behavior?
  1. Repricing is more frequent in high-inflation environments (Calvo models assume constant repricing)
  2. New data shows that prices change more than what usually assumed in Calvo models
  3. Constant probability of price-adjustment rules out any selection effect (firms that change prices may be the ones whose prices are most
GL build a model where the firm’s decision of reprice is along the two dimensions: size and time
Households

- Continuum of infinitely lived households each of which consumes a continuum of goods ($\Omega = P \times V$)

\[
c_t = \left[ \int_\Omega C_t(p)^{1 - (1/\varepsilon)} \phi_t(dp, dv) \right]^{\varepsilon/\varepsilon-1}
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- Preferences on leisure are linear
Optimization problem

- Preferences

\[
\text{Max}_{C_t(\cdot), l_t, \hat{m}_t} E \left[ \int_0^\infty e^{-\rho t} \left( \frac{1}{1-\gamma} c_t^{1-\gamma} - \alpha l_t + \log \left( \frac{\hat{m}_t}{P_t} \right) \right) \, dt \right]
\]
Optimization problem

- **Preferences**

\[
\text{Max}_{C_t(\cdot), l_t, \hat{m}_t} E \left[ \int_0^\infty e^{-\rho t} \left( \frac{1}{1 - \gamma} c_t^{1-\gamma} - \alpha l_t + \log \left( \frac{\hat{m}_t}{P_t} \right) \right) dt \right]
\]

- **Life-time budget constraint. (multiplier } \lambda)\)

\[
E \left[ \int_0^\infty Q_t \left( \int_{\Omega} pC_t(p) \phi_t(dp, dv) + R_t \hat{m}_t - w_t l_t - \Pi_t \right) dt \right] \leq m_0
\]
First-order conditions

\[ e^{-\rho t} c_t^{-\gamma} c_t^{1/\varepsilon} C_t(p)^{-1/\varepsilon} = \lambda Q_t p \quad (C_t(p)) \]

\[ \frac{e^{-\rho t}}{\hat{m}_t} = \lambda Q_t R_t \quad (\hat{m}_t) \]

\[ \alpha e^{-\rho t} = \lambda Q_t w_t \quad (l_t) \]

1. Let’s propose an equilibrium where \( R_t \) is constant
2. \( w_t \) inherits the stochastic properties of \( m_t \)

\[ w_t = \alpha R m_t \]

3. 
\[ d \log (m_t) = \mu dt + \sigma_m dZ_m \]
The production function exhibits CRS and productivity follows a mean reverting stochastic process

\[ C_t(p) = v_t l_t \]

\[ d \log(v_t) = -\eta \log(v_t) \, dt + \sigma_v \, dZ_v \]

Prices can be adjusted any time but firms face a menu cost denominated in terms of labor units.

From the optimization problem of the households, firms face the following demand schedule

\[ C_t(p) = \left( \frac{\alpha p}{w_t} \right)^{-\varepsilon} c_t^{1-\varepsilon\gamma} \]
Pricing strategy

• Price setting: Firms choose both a time length $T$ for their prices and an optimal new price $q$.
• Present value of a firm $\varphi(p, v, w, \phi_t)$
• $\phi_t$ joint distribution of $(p, v)$ across firms

$$\varphi(p, v, w, \phi_t) = \max_T E_t \left[ \left( \int_t^{t+T} Q_s C_s (p) \left( p - \frac{w_s}{v_s} \right) ds \right) + Q_T \cdot \max_q \left[ \varphi(q, v_{t+T}, w_{t+T}, \phi_{t+T}) - kw_{t+T} \right] \right]$$
Market clearing

- Markets: money, labor, final (consumption) good

\[
m_t = \hat{m}_t \\
l_t = \int_{\Omega} \frac{C_t(p)}{v} \phi_t(dp, dv) + k\Upsilon_t \\
C_t(p) = v_t l_t = \left(\frac{\alpha p}{w_t}\right)^{-\varepsilon} c_t^{1-\varepsilon\gamma}
\]

where \(\Upsilon_t = \Upsilon_t\) is the number of repricing firms.
First case: Only idiosyncratic shocks

- Both the money supply and the nominal wage have a drift of $\mu$ and variance of zero (deterministic).
- Conjecture an equilibrium where the distribution across firms is invariant ($\phi_t = \phi$) and then aggregate consumption is constant ($c_t = \bar{c}$).
- Using the scaling property, GL proposed a solution of the form

$$\varphi(p, \nu, w) = w \psi(x, \nu)$$

where $x = p/w$ and

$$\psi(x, \nu) = \max_T E \int_0^T e^{-\rho t} (\bar{c})^{1-\gamma} (\alpha x_t)^{-\varepsilon} \left( x_t - \frac{1}{v_t} \right) dt$$

$$+ e^{-\rho T} \cdot \max_{x'} \left[ \psi(x', \nu(T)) - k \right]$$
Fix-point solution

1. Conjecture $\bar{c}_i$
2. Solve for the value function $\psi (x, v)$
3. Find the policy function and the associate distribution of prices over the state of idiosyncratic shocks
4. Compute $\bar{c}_{i+1} = \left[ a^{1-\varepsilon} \int x^{1-\varepsilon} \phi_t (dx, dv; \bar{c}) \right]^{1/[1-\gamma(\varepsilon-1)]}$ and update your initial guess. Fix point $\bar{c}_i = \bar{c}_{i+1}$
Approximate the problem by a discrete time version with increments of size $h$ (for time) and state space $S = X \times V$

$$\psi (x, v) = \max \left\{ \Pi (x, v) \Delta t + e^{-r \Delta t} \sum_{x',v'} \pi (x', v' \mid x, v) \psi (x', v'), \right. $$

$$\left. \max_{\xi} [\Pi (\xi, v) \Delta t + e^{-r \Delta t} \sum_{x',v'} \pi (x', v' \mid \xi, v) \psi (x', v') - k] \right\}$$

where $\Pi (x, v) = (\bar{c})^{1-\varepsilon} (\alpha x_t)^{-\varepsilon} (x - \frac{1}{v})$ and $\pi$ is a transition function define on $S \times S$ (Markov-chain probabilities)
## Calibration

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>target/description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$  discount rate</td>
<td>0.4</td>
<td>interest rates</td>
</tr>
<tr>
<td>$\gamma$ risk aversion</td>
<td>2</td>
<td>usual RBC</td>
</tr>
<tr>
<td>$\varepsilon$ elasticity of sub.</td>
<td>7</td>
<td>markups of 16%</td>
</tr>
<tr>
<td>$\alpha$ disutility labor</td>
<td>6</td>
<td>$L=37%$</td>
</tr>
<tr>
<td>$\mu$ drift inflation process</td>
<td>0.0064</td>
<td>quarterly inflation rate</td>
</tr>
<tr>
<td>$k$ menu cost</td>
<td>0.0025</td>
<td>frequency of price changes</td>
</tr>
<tr>
<td>$n$ productivity shock</td>
<td>0.55</td>
<td>log-price increase over new prices</td>
</tr>
<tr>
<td>$\sigma_v^2$ variance prd. shock</td>
<td>0.011</td>
<td>standard deviation new prices</td>
</tr>
</tbody>
</table>
Qualitative features of this case
Results

Fig. 3.—Fraction of prices changed each month
Adding aggregate shocks

One-time jump of money growth rate

1. Jump from $\mu$ to $\mu(1 + h)$ leads to a jump of wages by the same amount. Output increases at most by $(1 + h)^{1/\gamma}$

2. The fraction of firms that reprice is higher. Firms that reprice are the ones that have the most incentives to reprice (change in prices is large)
Self-selection: comparison between Calvo and GL

Fig. 6.—Price adjustment in menu cost and Calvo models. a, Price adjustment before aggregate shock. b, Price adjustment after aggregate shock.
Hazard functions

- Absent of aggregate shocks, the model predicts a downward slopping hazard function. Recent updated prices are most likely to come from high-productive firms (high-sensitive to price misalignments) and have high probability to change again in the near future. Old prices most likely come from low-productive firms (with large inaction bands) and are unlikely to change. (flatter curve)

- Elasticity of substitution: the larger the elasticity the less is the difference in the inaction band between high and low-productive firms

- If aggregate shocks are present the model predicts upward slopping hazard function, provided aggregate shocks are persistent and idiosyncratic innovations fade out and have relative small variances.
Macro implications

One time shock
Macro implications II
Two aggregate shocks

- Monetary fluctuations can account for less than 10% of the observed fluctuations in output (simulations of 40 quarters of data)
- Using the simulated series, GL run the following regression:

$$\log \left( y_t^Q \right) = \alpha + \beta \left[ \log \left( w_t^Q \right) - \log \left( w_{t-1}^Q \right) \right]$$

- The estimation of the parameters suggests that the Phillips-Curve holds ($\hat{\beta}$ is 0.049) but the effect is quite small
Three "New" Features of the Distribution of Price Changes

Using scanner price data collected in retail stores and excluding sales, Midrigan documents:

1. Large number of small price changes
2. Fat Tails (excess Kurtosis)
3. Adjustment in tandem within a store

Extending a standard menu cost model to a multi-product setting with economies of scope in technology of price adjustment can replicate these facts and generate larger aggregate fluctuations than standard menu cost economies.
Multiproduct Menu Cost Model

Two main departures from GL:

1. Multiproduct firms (vs single product firms) that face a fixed cost of changing its entire menu of prices but, conditional on paying this cost, zero marginal cost of resetting any given price on the menu

2. Leptokurtic shocks (vs Gaussian shocks)

These two features determine that the selection effect present in GL is much weaker in here and thus responsiveness of aggregate price levels to monetary shocks is reduced generating larger aggregate fluctuations
Quantitative Results: Parameters

- Preferences (Hansen 1985)
  \[ U(c, n) = \log(c) - \psi n \]

- Parameters (Benchmark)

<table>
<thead>
<tr>
<th>Parameters not explicitly solved for:</th>
</tr>
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<tbody>
<tr>
<td>Common:</td>
</tr>
<tr>
<td>( \beta ) discount factor</td>
</tr>
<tr>
<td>( \delta ) persistence of money shocks</td>
</tr>
<tr>
<td>( \sigma^2 ) variance of money shocks</td>
</tr>
<tr>
<td>( \psi ) marginal disutility from work</td>
</tr>
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<td>( \theta ) elasticity of substitution</td>
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<tr>
<td>( \sigma^2 ) variance of technology shocks</td>
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<tr>
<td>( \xi ) menu cost, % of SS labor bill</td>
</tr>
<tr>
<td>( \alpha_1 ) Beta(( \alpha_1, \alpha_2 ))</td>
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<tr>
<td>( \rho ) persistence of technology shocks</td>
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<tr>
<td>( \varepsilon_{\text{max}} ) upper bound of shock distribution</td>
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## Quantitative Results: Calibration Targets

- **Calibration Targets**

<table>
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<tr>
<th>Moments</th>
<th>Targets</th>
<th>Benchmark</th>
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<tbody>
<tr>
<td>Duration of price spells</td>
<td>4.5</td>
<td>4.6</td>
</tr>
<tr>
<td>mean ($</td>
<td>\Delta p</td>
<td>$), %</td>
</tr>
<tr>
<td>std ($\Delta p$), %</td>
<td>12.0</td>
<td>11.8</td>
</tr>
<tr>
<td>kurtosis($\Delta p$)</td>
<td>4.5</td>
<td>4.3</td>
</tr>
<tr>
<td>$Pr(</td>
<td>\Delta p</td>
<td>&lt; \text{mean}(</td>
</tr>
<tr>
<td>Deviance ratio: 24 months</td>
<td>1.02</td>
<td>1.01</td>
</tr>
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</table>
Small price changes will arise in equilibrium whenever at least one of the firm’s two prices is hit by a sufficiently large shock.

Firms are more willing to adjust prices in periods when their technology is higher ($\sim$GL).
### Calibration

**Matching Micro Features of the Data**

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<td>mean (</td>
<td>Δp</td>
<td>), %</td>
<td>9.0</td>
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<td>std (Δp), %</td>
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<td>9.3</td>
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<td>12.5</td>
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<td>kurtosis(Δp)</td>
<td>4.5</td>
<td>4.3</td>
<td>1.3</td>
<td>1.7</td>
<td>4.0</td>
</tr>
<tr>
<td>Pr(</td>
<td>Δp</td>
<td>&lt;mean(</td>
<td>Δp</td>
<td>)/2)</td>
<td>0.30</td>
</tr>
<tr>
<td>Deviance ratio: 24 months</td>
<td>1.02</td>
<td>1.01</td>
<td>0.84</td>
<td>0.89</td>
<td>1.12</td>
</tr>
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</table>
Aggregate Implications

Interactions in the costs of price adjustment + leptokurtic shocks are thus a necessary ingredient of a model capable of reproducing the microeconomic evidence and generating sizable business cycle fluctuations from monetary disturbances.
Real Effects from Money Shocks: MP vs GL

Impulse Response to 1% increase in money growth rate
Aggregate price level response ($\pi$):

$$\pi = \log \frac{P}{P_{-1}}$$

$$= \text{fraction of adjusters} \times \text{mean price } \Delta \text{ cond. on a price } \Delta$$

**Extensive Margin**

**Intensive Margin**
Explaining the "Dampened" Price Response: Weaker Selection Effect

- Selection effect in standard menu cost models (GL)
- Strength of selection effect, density of desired price changes and adjustment hazards:

![Graphs showing benchmark setup and Golosov-Lucas'03 setup](image-url)
Explaining the "Dampened" Price Response: Selection Effect and Distribution of Price Changes

Effect of money shock on the distribution of non-zero price changes

![Graphs showing the distribution of price changes](image-url)
Counterfactuals

1. Calvo-Timing: Self-Selection and Synchronization are shut down
2. No Self-Selection

→ Self Selection and Front Loading
Conclusions

- Standard single-product state-dependent pricing models are inconsistent with large number of small price changes and excess kurtosis of price changes found in the data.
- These facts can be reconciled with state-dependent models if multi-product firms face interactions in the costs of adjusting.
- This model can generate business cycle fluctuations from nominal disturbances that are almost as large as in Calvo-style time-dependent models. A key feature of the calibration, the leptokurtic distribution of idiosyncratic disturbances, together with the assumption of economies of scale in the price adjustment technology, implies that the selection effect that plays an important role in standard menu cost economies is much weaker in this setup.